FLUIDS: Liquids and Gases

A. OVERVIEW: Fluids exert 3 different types of forces on objects

1) Buoyant forces: all objects immersed in a fluid are subject to an upward force equal to the weight of the fluid displaced by the object.

2) Drag **forces:** any object moving relative to a fluid is subject. The magnitude of this force depends upon the surface area of the object, the density of the fluid, and the speed at which the object moves relative to the fluid. Because the drag forces change as an object speeds up or slows down, this leads to non-constant acceleration, which requires calculus to describe mathematically. We won't do any calculations involving drag forces.

3) Bernoulli **effect forces:** When a fluid flows at different speeds over different surfaces of an object, there is a difference in Pressure on the 2 surfaces, which leads to an unbalanced force. This force may sometimes be referred to as "Lift" (particularly in the case of the forces exerted on airplane wings) but it need not be an upward force.

B. Fluids are described by 2 intrinsic properties: **DENSITY** and **PRESSURE**

1. DENSITY

Density is not too difficult to understand. It is simply defined as the mass per unit volume of a substance or of an object:

$$\rho = m/v$$

In Chemistry, the preferred unit of density is usually g/cm^3 or g/ml (you may recall that the density of water is 1 g/ml). In Physics, **the preferred unit of density is kg/m^3**, so that no conversions are needed to calculated masses in kg or forces in Newtons. There are 1000 g in a kg but there are 1,000,000 cm^3 (100^3) in a m^3. So **the density of water is 1000 kg/m^3**.

(note: in case it ever comes up, 1 liter = 1 dm^3 = 1/1000 m^3, so 1 m^3 = 1000 l)

e.g.1. Calculate the weight of a cube of gold (ρ_{gold}= 1900 kg/m^3) that is 20 cm on a side.

2. PRESSURE

Pressure is a more difficult concept than Density, as it involves forces that may be difficult to visualize. Pressure is simply defined as Force per unit Area:

$$P = F/A$$

e.g.2. A car with mass 2000 kg sits on 4 tires, each of which has a "footprint" of 200 cm^2. Calculate the pressure exerted by the car on the tires.

Fluids exert pressure on all objects within them, and these forces can be rather large. The pressure exerted by the atmosphere on an average day, for example, is around 14.7 lbs/in^2. Approximating the top of the average person's head as a circle of radius 4 inches, the downward force exerted by the atmosphere on your head is

$$14.7 \text{ lbs/in}^2 * \pi * 4 \text{ in}^2 = \text{around } 750 \text{ lbs!}$$

This pressure does not crush your skull because your skull is filled with fluids of its own, which exert equal pressure outward. Because atmospheric pressure is usually balanced on all sides of an object, we rarely notice it at all. Where the effects of pressure become quite noticeable is when pressure can be greatly reduced on one side of an object.

→ VACUUM PUMP DEMOS

e.g.3. Calculate the total downward force of the atmosphere acting on the top of a desk that measures 25 in X 16 in. Why is the desk not crushed?

A. **Hydraulics and Pascal's Principle**

It is an experimental fact that while fluid pressure may be different at different locations within a fluid, at a given location a fluid exerts equal pressure in all directions. This is sometimes referred to as "Pascal's Principle". Another important aspect of Pascal's principle is that if pressure in a confined fluid is increased (by the application of some external force), then the pressure within the fluid increases by the same amount throughout the fluid.

area
1 square inch
total force = 10 pounds

area
80 square inches
total force = 800 pounds
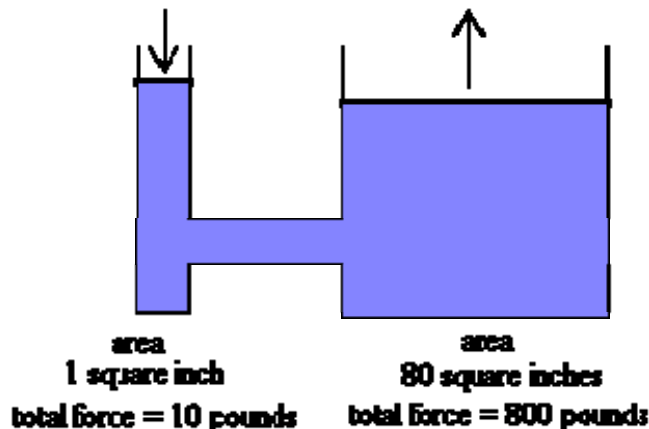
Hydraulic devices make use of Pascal's principle by using small forces to generate much larger ones. A piston of small area is pressed into a fluid at a point of narrow area within a container, which increases the pressure within the fluid by an amount equal to F_{in}/A_{iin}. The pressure within the entire fluid is increased by this same amount. The force applied to the large piston is then equal to the increase in Pressure multiplied by the larger area →

$$F_{out} = (F_{in}/A_{in})A_{out}$$

The pressure is thus effectively multiplied by the ratio A_{out}/A_{in}, which can be made very large. The tradeoff, of course, is that the larger piston moves by an amount that is much smaller than the smaller piston. An ideal hydraulic system would be 100% energy efficient, so that the work done by the smaller piston would all be transferred into work done by the larger piston, thus

$$W_{out} = W_{in} \text{ and } \qquad F_{out}\Delta x_{in} = F_{in}\Delta x_{in}$$

In practice, of course, with real fluids and real machines, W_{out} will always be a little less than W_{in}.

e.g. 4. by pushing with your foot on the brake pedal, a force of 200 N is applied to the brake master cylinder, which has an area of 4 cm^2. This causes force to be applied at 4 brake pads, each of which touches an area of 100 cm^2. Assuming, 100 % efficiency, how much total braking force is applied to the wheels?

If the master cylinder moves a distance of 10 cm, how far does each brake pad move?

B. **PRESSURE GAUGES:**

The pressure inside of a closed container can be measure using a Pressure Gauge. Most pressure gauges do not actually measure the absolute pressure though (if you had a tire pressure gauge in this room right now, it would likely read zero!). Pressure gauges normally measure the difference between the absolute pressure inside the container and the gauge pressure.

$$P_{gauge} = P_{absolute} - P_{atmospheric}$$

E.g. 5. your car tires tell you to inflate to 35 psi. What is the actual pressure inside the tires when inflated to 35 psi?

Although psi are a common pressure unit in the everyday world, in Physics the preferred pressure unit is the N/m2, which is also known as the "Pascal" (Pa). The Pascal is a very small unit of pressure: on an average day, the pressure in the atmosphere is around 101,000 Pa. For convenience sake, this is sometimes referred to as "1 atmosphere". One other pressure unit with which you are probably familiar from Chemistry is the mm Hg, which relates to the height of a column of mercury that can be supported by the atmosphere (see next section for more on that subject)

101, 000 N/m^2 = 101,000 Pa = 101 kPa = 1 Atm = 760 mm Hg

C. **ATMOSPERHIC PRESSURE:**

How is atmospheric pressure itself measured? By a device you have doubtless heard of called a barometer. A barometer is essentially an inverted tube filled with mercury that has a vacuum at the top and sits in a pool of mercury open to the atmosphere at the bottom:

Conceptually, the harder the atmosphere pushes down on the mercury surface, the higher the column of mercury that will be supported. More quantitatively, the relationship between the pressure in the atmosphere and the height of the column can be understood by drawing a free-body diagram for the column of mercury

$W = \rho V g$ $P*A$
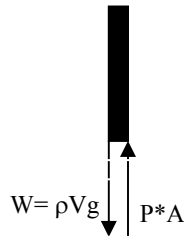
The column of mercury is pulled downward by gravity and pushed upward by the pressure at the surface of the liquid. The weight of the column can be calculated as mass * acceleration due to gravity and the mass of the column can be calculated as the density of mercury times the volume of the column. Thus

$$W = \rho V g$$

Since the volume of the column simply equals the height of the column * the cross-sectional area of the column, this can be further expanded to

$$W = \rho h A g$$

Since by definition, $P = F/A$, the upward force exerted on the column can be calculated as the atmospheric pressure times the area.

$$F = PA$$

Since the column of mercury is at rest, it follows that $F = W$, so $\rho h A g = PA$.

Canceling out the "A"s, we see that the Pressure is given by $P = \rho h g$.

Rearranged, we can see that the height of the column will be given by $h = P/\rho g$

Plugging in values for mercury, $h = 101{,}000 \ \text{N/m}^2 / \ (13{,}600 \ \text{kg/m}^3)(9.8 \ \text{m/s}^2)$

$$= 0.76 \ \frac{\text{N/m}^2}{\text{N/m}^3} = 0.76 \ \text{m (or 760 mm)}$$

Could you make a barometer out of a liquid other than mercury? Sure you could. But since the height of the column is inversely proportional to the density of the liquid, mercury is a natural choice because it is the densest room temperature liquid available. Water is 13.6 times less dense than mercury. So a water barometer would have to be 0.76 m * 13.6 = 10.3 meters tall! This is about 34 feet. Since the atmosphere cannot support a column of water taller than this, this implies by the way that it is impossible to suction water vertically more than 34 feet, Even Superman could not drink from a straw more than 34 feet above a water surface!

3. THE PRESSURE-DEPTH RELATIONSHIP: $P = P_{atm} + \rho gh$

What causes atmospheric pressure? Some insight into this can be gained from picturing a column of air, area A, stretching from just above sea level all the way to the top of the atmosphere.

A free-body diagram for this column of air would show that it has weight, which acts downward, and has pressure from the sea-level air underneath it, and that the forces are balanced. Thus,
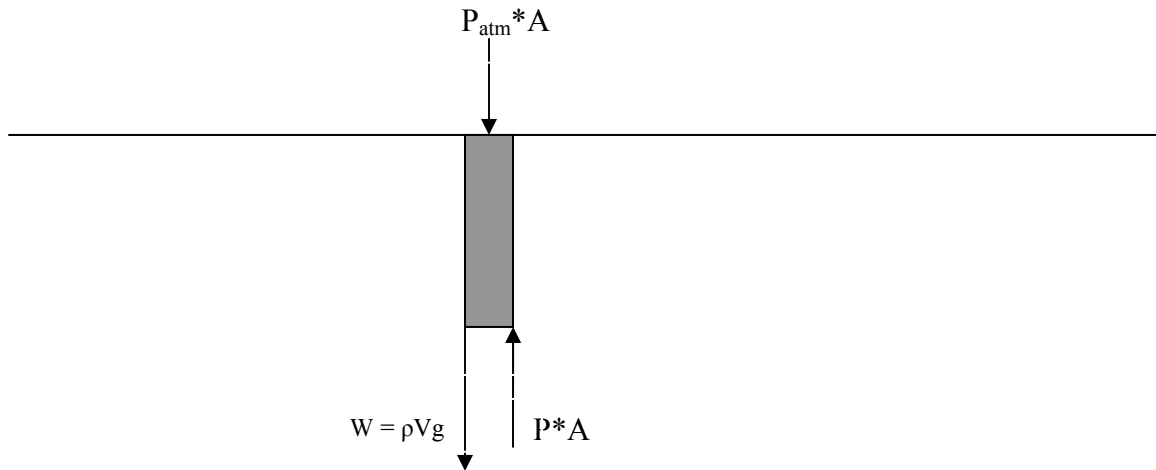
$$W_{column} = PA$$

So the pressure in the atmosphere equals the weight per unit area of a column of air stretching all the way to the top of the atmosphere.

Since the mass of the column of air could be calculated as the density of the column of air times its volume (which equals its height * its cross-sectional area), this can be rewritten as

$$\rho h A g = PA$$

This expression is not useful however (unless you know calculus…), because the density of the atmosphere diminishes continuously from sea level on up.

Unlike gases, however, liquids are for practical purposes incompressible (i.e. the density does not vary). Thus if we perform a similar analysis below the surface of a liquid,

$$P_{atm}*A$$

$$W = \rho Vg \qquad P*A$$

The forces on the column of liquid are the air pushing down from above, the liquid beneath the column pushing up from underneath, and the weight of the column of liquid. Since the forces are balanced,

$$P_{atmosphere}*A + \rho hag = PA$$

Canceling out the "A"s and rearranging slightly yields the pressure-depth relationship: $\mathbf{P = P_{atm} + \rho gh}$

e.g.6: How far down below the surface of a lake do you need to go before the pressure becomes twice atmospheric pressure? ($\rho_{water} = 1000$ kg/m^3)

## 4. BUOYANT FORCES: $\mathbf{F_B = \rho_{fluid}gV_{fluid\ displaced}}$

All objects immersed in a fluid feel an upward buoyant force equal to the density of the fluid * the rate of acceleration due to gravity * the volume of fluid displaced by the object.

(NOTE: if an object is fully submerged in a liquid, then the volume of liquid displaced = the volume of the object but if it is floating at the surface, then the volume of fluid liquid displaced = the volume of the object which is underwater)

The product of ρ_{fluid} , g, and $V_{fluid\ displaced}$ can also be thought of simply as the weight of fluid displaced by the object. That the buoyant force on an object is equal to the weight of fluid it displaces was first discovered by an ancient Greek mathematician named Archimedes, and is thus referred to as Archimedes Principle : $F_B = W_{fluid\ displaced}$

But what causes this upward force? The buoyant force equation can actually be derived from a careful analysis of the pressure-depth relationship: $P = P_{atm} + \rho gh$

Picture an object immersed in a liquid:
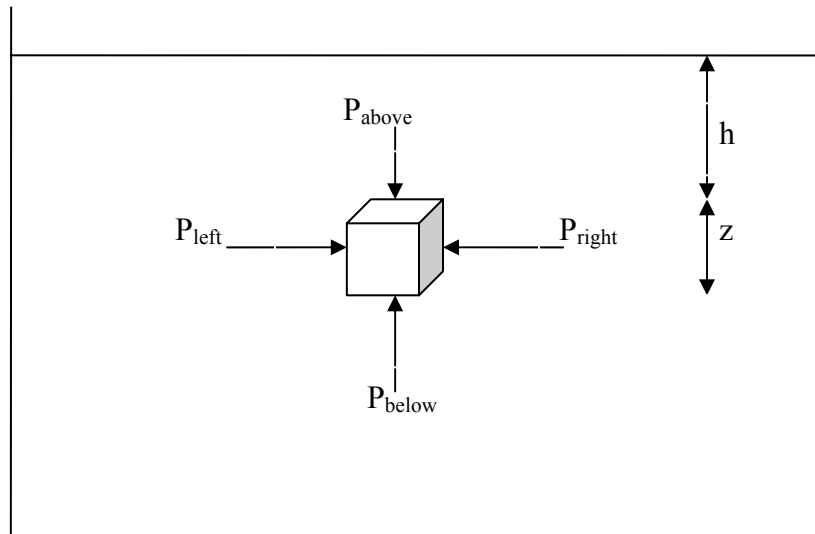
The object has a height of "z", cross-sectional area A, and is "h" meters below the surface of the liquid, which has density ρ_{fluid}.

The object is subject to Pressure on all sides. The pressure from the left = the pressure from the right. But the pressure from above ≠ the pressure from below.

In fact, the pressure from above = $P_{atm} + \rho_{fluid}gh$ while the pressure from below = $P_{atm} + \rho_{fluid}g(h+z)$

So the force pushing down from above = $(P_{atm} + \rho_{fluid}gh)A$
while the force from below = $(P_{atm} + \rho_{fluid}g(h+z))A$

Thus, there is a net upward force of $(P_{atm} + \rho_{fluid}g(h+z))A - (P_{atm} + \rho_{fluid}gh)A = \rho_{fluid}gzA$,

Since zA = the volume of the object (and also of the fluid displaced in this case), the net upward force can also be written as

$$F_{upward} = \rho_{fluid}gV_{fluid\ displaed}$$

We call this net upward force due to differences in pressure a "Buoyant force."

e.g. 7. a stone statue of mass 200 kg and average density 7000 kg/m³ sits at the bottom of the ocean, 500 m underwater. The density of ocean water is 1030 kg/m³. The statue is to be lifted up at constant velocity by a cable. What tension will this require?

e.g .8. a wood block 1/3 as dense as water with dimensions 10 cm * 20 cm * 30 cm high floats at the surface of a swimming pool. How far up is the water line from the bottom of the block?

e.g. 9. A steel sphere of radius 10 cm and density 7800 kg/m^3 is released just below the surface of a swimming pool. What will be its acceleration at the moment that it is released?

5. **HYDRODYNAMICS**

We move now to the topic of fluids in motion: hydrodynamics. Note that this is an extremely complicated (though highly practical) subject and we will only touch upon the very tip of the great iceberg of fluid dynamics.

For starters, fluid flow can be either smooth ("laminar") or rough ("turbulent"). We consider only smooth flows. Turbulent flow is much more complicated and currently not all that well understood.

Secondly, real fluids have some internal resistance to flow ("viscosity") and are to varying extents compressible. We consider only "inviscid" and "incompressible" fluids. Considering viscosity and compressibility makes all analyses far more complex.

With the disclaimers that we consider only smooth flows of inviscid, incompressible fluids, I now state to you that these flows can be described by 2 major equations:

    The Continuity Equation: $\mathbf{A_1v_1 = A_2v_2}$ and

    The Bernoulli Equation: $\mathbf{P_1 + 1/2\ \rho v_1^2 + \rho gh_1 = P_2 + 1/2\rho v_2 2 + \rho gh_2}$

These equations are often applied to fluids confined to containers (usually pipes) but may also be applied to freely flowing fluids.

A. The continuity equation: $\mathbf{A_1v_1 = A_2v_2}$

The continuity equation is generally applied to fluids constrained to a container and arises from the idea of conservation of mass: as much fluid flows into a container as flows.

The product A*v is called the volume flow rate, normally quantified in m^3/s. Unless the level of fluid in a container is either rising or falling, the volume of fluid flowing into the container must = the volume of fluid flowing out, thus Flow Rate$_1$ = Flow Rate$_2$ and $\mathbf{A_1v_1 = A_2v_2}$

       A_1

A_2

v_1

v_2
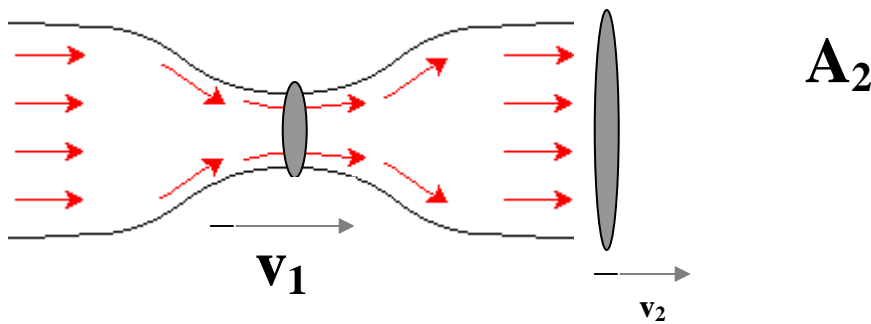
Thus as the pipe gets narrower, the liquid must speed up to maintain a constant flow rate. Those of you who have done any canoeing, rafting, or kayaking will be familiar with the idea that rivers flow fastest where they are most narrow and/or most shallow and slow down considerably where they are deeper and wider (ever heard the expression "still waters run deep"?). This is described nicely by the continuity equation.

e.g. 10. The diameter of a pipe shrinks from 4 in to 3 in over a distance of 8 in. When the level of water in the pipe is steady, how many times faster will the water be flowing in the narrow section than in the wider section?

B. The Bernoulli equation: $P_1 + 1/2\ \rho v_1^2 + \rho g h_1 = P_2 + 1/2 \rho v_2^2 + \rho g h_2$

      (alternately, the Bernoulli equation can be written as $P + 1/2\ \rho v^2 + \rho g h = \textbf{constant}$)

The Bernoulli equation can be used to solve for Pressure differences in which fluids are changing heights and/or speeds. It can also be written as

Often the change in height is considered negligible, for example, in an airplane wing, the difference in height between the top and bottom of the wing is very small while the difference in flow speeds is quite large so the $\rho g h$ would likely be ignored and the Bernoulli equation simplifies to

$$P_1 + 1/2\ \rho v_1^2 = P_2 + 1/2 \rho v_2^2 \quad (\text{or } P + 1/2\ \rho v^2 = \text{constant})$$

A careful look at the equation shows that as the velocity of a fluid increases, the Pressure must decrease. This is a somewhat counter-intuitive result, as we usually think of fast flowing fluids as exerting large forces. The pressure in a fluid, however, is the force that it exerts in all directions, not the forward force that the fluid would exert.

The inverse relationship between Pressure and fluid velocity is referred to descriptively as "Bernoulli's Principle", and can be invoked to explain all kinds of everyday occurrences:

      e.g. leaves being picked up from the ground by a gust of wind
            the fact that you can "lift" a piece of paper by blowing strongly over the top of it
            the shower curtain creeping in on you when you turn the water on full blast

the sideways force exerted in the wake of a fast-moving truck or train
an umbrella turning upward on a windy day
all manner of curving balls
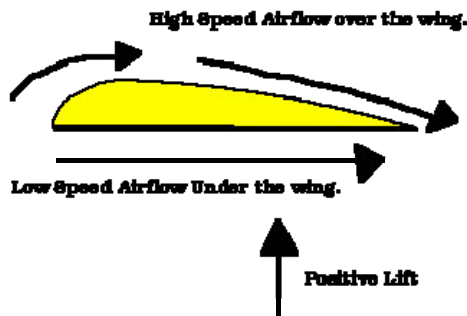
Surfaces that are built in such a way as to cause air to flow faster over one surface than another are called "airfoils". Such surfaces include airplane wings, bird wings, Frisbees, sails, and more. Note that airfoils can also be thought of simply as objects which direct air upward. In pushing air upward, the airfoil itself is pushed downward by the air, in accordance with Newton's 3rd law. All examples of Bernoulli's principle can alternatively be understood in this way.

"Spoilers" on the back of sports cars are upside-down airfoils, built to cause a downward force on the back of the car for better traction. (Actually, at normal highway speeds, this force is insignificantly small, so most spoilers are really more of a fashion statement than anything else).

Some racecars, however, are built so that the entire body of the car acts as an upside-down airfoil. At racing speeds, these cars generate downward forces due to airflow that are actually greater than the cars' weight. So at high speeds, these cars could actually drive upside down!

e.g. 11: Air flows over the top surface of an airplane wing at a speed of 340 m/s and over the bottom surface at a speed of 290 m/s. The cross-sectional area of the wing is 80 m^2. Calculate the lift force on the wing, ignoring the change in height between the top and bottom.

Consider that the change in height between the top and bottom is 1 meter. How does this affect your answer?

COMBINING THE CONTINUITY AND BERNOULLI EQUATIONS:

e.g. 12: Household pipes:

Hot water is pumped at a speed of 0.50 m/s through a 4.0 cm diameter pipe in the basement of a house under pressure of 300 kPa. What will be the pressure in a 2.6 cm diameter pipe on the second floor 5.0 m above?

e.g. 13. Speed of Efflux

Water drains from a large reservoir of cross-sectional area **A** through a small hole of cross-sectional area. The hole is a distance **h** beneath the top surface of the liquid. Assuming that both the top and the spout are open to the atmosphere and that **A >> a**, calculate the speed at which the water comes out the hole (called the "speed of efflux")

e.g 14. The venturi meter:

A venturi tube is a pipe with a narrow constriction that can be used to measure the flow rates of liquids and gases.
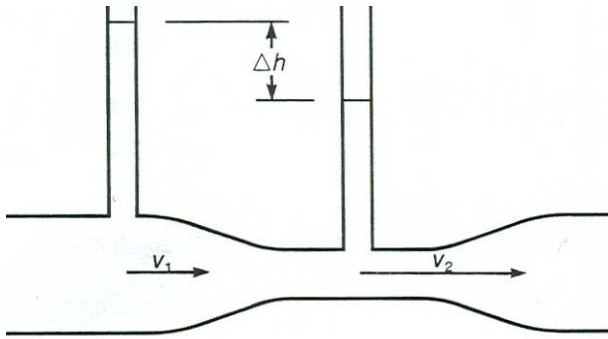
The tube is inserted into a pipe and the difference in pressure between the thick and thin sections of the pipe can be used to determine flow rates. If a venturi tube of $A_1 = 1.0 \ cm^2$ and $A_2 = 3.0 \ cm^2$ measures a difference in height of 5 cm (the liquid is water),

a) what is the pressure difference between the thin and thick sections of the tube?

b) what are the flow velocities in each section of the tube?

c) What is the flow velocity and the pressure in the pipe itself (not shown), which has cross-sectional area 5 cm^2?

e.g. 15. A plastic tank 20 meters wide by 25 meters long by 3 meters deep drains through a hole 20 cm above the bottom of the tank which has a cross sectional area of 25 cm^2. It begins to rain, with water coming down at a rate of 5 cm/hour. Assuming that the rain continues to fall at this rate, how high will the water level in the tank get before reaching equilibrium with the tank draining as fast as it fills?