On the Laws of Induction:
Faraday's Flux Rule versus the Lorentz Force

<u>Part 1:</u>

Faraday's experimental observations led to the flux rule for circuits, i.e., the discovery that in a region where the magnetic field is changing in time, electric fields are generated. It can be stated mathematically as

$$\oint_{\Gamma} \vec{E} \cdot d\vec{s} = -\frac{\partial \phi_M}{\partial t} ,$$

where Γ is any closed circuit and ϕ_M is the magnetic flux through the surface bounded by Γ. The integral on the left is the emf, and that on the right is the negative of the flux linked by the circuit.

The non-conservative nature of the induced electric potential can be regarded as a fictitious voltage in Faraday's law (in the same line of thinking as fictitious forces are used in non-inertial frames of reference). In viewing motion, often we put ourselves in the moving frame of reference, if the motion looks easier in that particular frame. This scheme avoids the introduction of voltmeters in a multiple connected region.

■ FOR FURTHER THOUGHT NONCONSERVATIVE ELECTRIC FIELDS

The nonconservative nature of the induced electric field is strikingly demonstrated if you attempt to measure potential differences in nonconservative fields. In Chapter 25, we defined potential difference as the work required to move a unit charge between two points, and stressed that this work is independent of path for a conservative field. But when the field is nonconservative, the work is not independent of path, and the concept of potential becomes ambiguous.

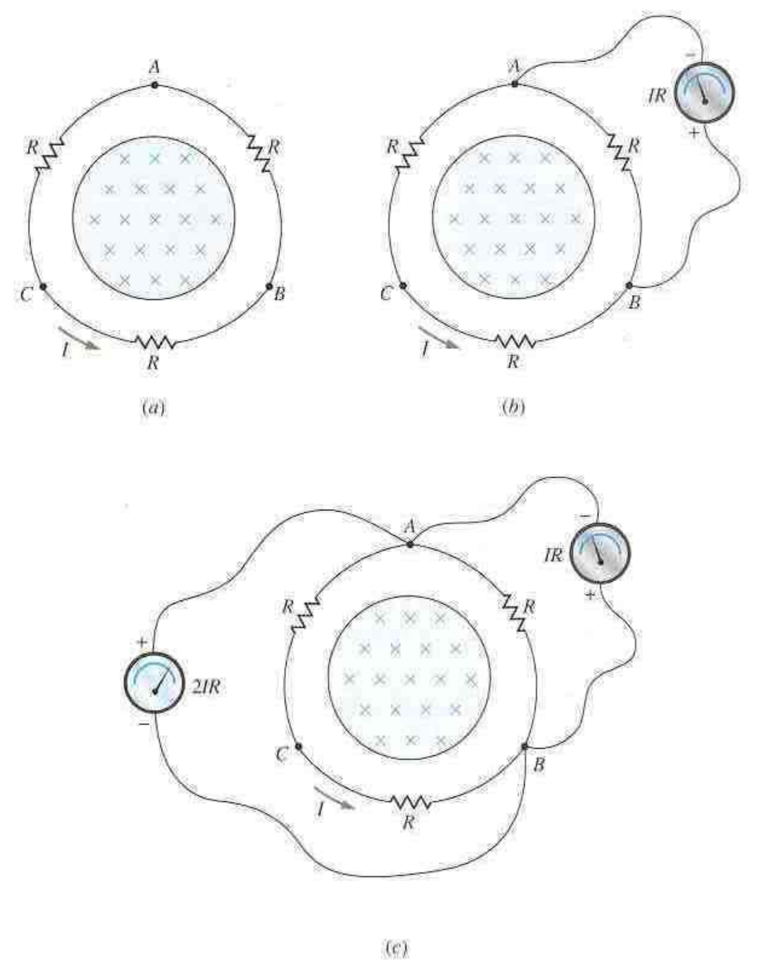
Figure 31-31 shows an end view of a long solenoid surrounded by three identical resistors bent into circular arcs. If the solenoid current is increasing, an induced electric field appears in the resistors, and drives a current *I* in the counterclockwise direction.

Because they have the same resistance and carry the same current, the potential difference across each resistor should be the same. We could try to measure the potential difference across one resistor, for example, by connecting a voltmeter as shown in Fig. 31-31*b*. With current *I* flowing through the resistance *R*, this meter reads *IR*. Since the current flows counterclockwise, we must connect the positive voltmeter terminal to point *B*.

But now try to measure the potential difference across the other two resistors together, as in Fig. 31-31*c*. With the current *I* flowing through the total resistance 2*R*, the meter now reads 2*IR*. We have two voltmeters with their terminals connected to the same points, and yet they indicate different voltages. Not only are the magnitudes of the voltages different, but even their polarities differ. How can this be? We are experiencing the nonconservative nature of the induced electric field. The two voltmeters are positioned differently with respect to the changing magnetic flux, so they sample different regions of the induced electric field. Even though the meters are connected to the same points, they measure the line integral of the induced electric field over *different* paths, and so they do not read the same voltage.*

*For a fascinating discussion of voltage measurement in induced fields, see R. Romer, "What do 'Voltmeters' Measure?: Faraday's Law in a Multiply Connected Region," *American Journal of Physics*, vol. 50, no. 12, pp. 1089–1091 (December 1982).

FIGURE 31-31 (*a*) End view of a long solenoid surrounded by three resistors in series. A changing magnetic field in the solenoid induces an emf in the resistors, and the same current *I* flows through each. (*b*) A voltmeter connected between points *A* and *B* indicates the voltage *IR* across one resistor. (*c*) A second voltmeter connected to the *same points* indicates voltage 2*IR* with the opposite sign.



References:

1. Richard Wolfson and Jay M. Pasachoff, "Physics for Scientists and Engineers" (Harper-Collins, NY, 2nd edition, 1995). See page 801.
2. R. Romer, “What do voltmeters measure?: Faraday’s Law of Multiple Connected Region”, American Journal of Physics, 50, 1089-1091 (1982).

Part 2:

What is the force acting on a stationary, point-like charge in the presence of a time-varying magnetic field produced by a coil?

$$F = q\frac{\partial}{\partial t}\left(\vec{r}\times\vec{B}\right) = q\vec{v}\times\vec{B} + q\vec{r}\times\frac{\partial\vec{B}}{\partial t}$$

If the charge is stationary then:

$$F = q\vec{r} \times \frac{\partial \vec{B}}{\partial t}$$

where $\vec{r}$ represents the position of the charge with respect to the center of the coil.